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Math 1010

Optimizing an Advertising Campaign

Math 1010 Intermediate Algebra Group Project

Background Information:

Linear Programming is a technique used for optimization of a real-world situation. Examples of optimization include maximizing the number of items that can be manufactured or minimizing the cost of production. The equation that represents the quantity to be optimized is called the objective function, since the objective of the process is to optimize the value. In this project the objective is to maximize the number of people who will be reached by an advertising campaign.

The objective is subject to limitations or constraints that are represented by inequalities. Limitations on the number of items that can be produced, the number of hours that workers are available, and the amount of land a farmer has for crops are examples of constraints that can be represented using inequalities. Broadcasting an infinite number of advertisements is not a realistic goal. In this project one of the constraints will be based on an advertising budget.

Graphing the system of inequalities based on the constraints provides a visual representation of the possible solutions to the problem. If the graph is a closed region, it can be shown that the values that optimize the objective function will occur at one of the "corners" of the region.

The Problem:

In this project your group will solve the following situation:

A local business plans on advertising their new product by purchasing advertisements on the radio and on TV. The business plans to purchase at least 60 total ads and they want to have at least twice as many TV ads as radio ads. Radio ads cost $20 each and TV ads cost $80 each. The advertising budget is $4320. It is estimated that each radio ad will be heard by 2000 listeners and each TV ad will be seen by 1500 people. How many of each type of ad should be purchased to maximize the number of people who will be reached by the advertisements?

Modeling the Problem:

Let X be the number of radio ads that are purchased and Y be the number of TV ads.

1. Write down a linear inequality for the total number of desired ads.

$$x+y \geq 60$$

2. Write down a linear inequality for the cost of the ads.

$$20x+80y \leq 4320$$

$$20x+80y \leq 4320$$
$$80y \leq -20x+4320$$
$$y \leq -\frac{1}{4}x+54$$
$$(0,54)$$

3. Recall that the business wants at least twice as many TV ads as radio ads. Write down a linear inequality that expresses this fact.

$$X \geq 2Y$$

4. There are two more constraints that must be met. These relate to the fact that there cannot be s negative numbers of advertisements. Write the two inequalities that model these constraints:

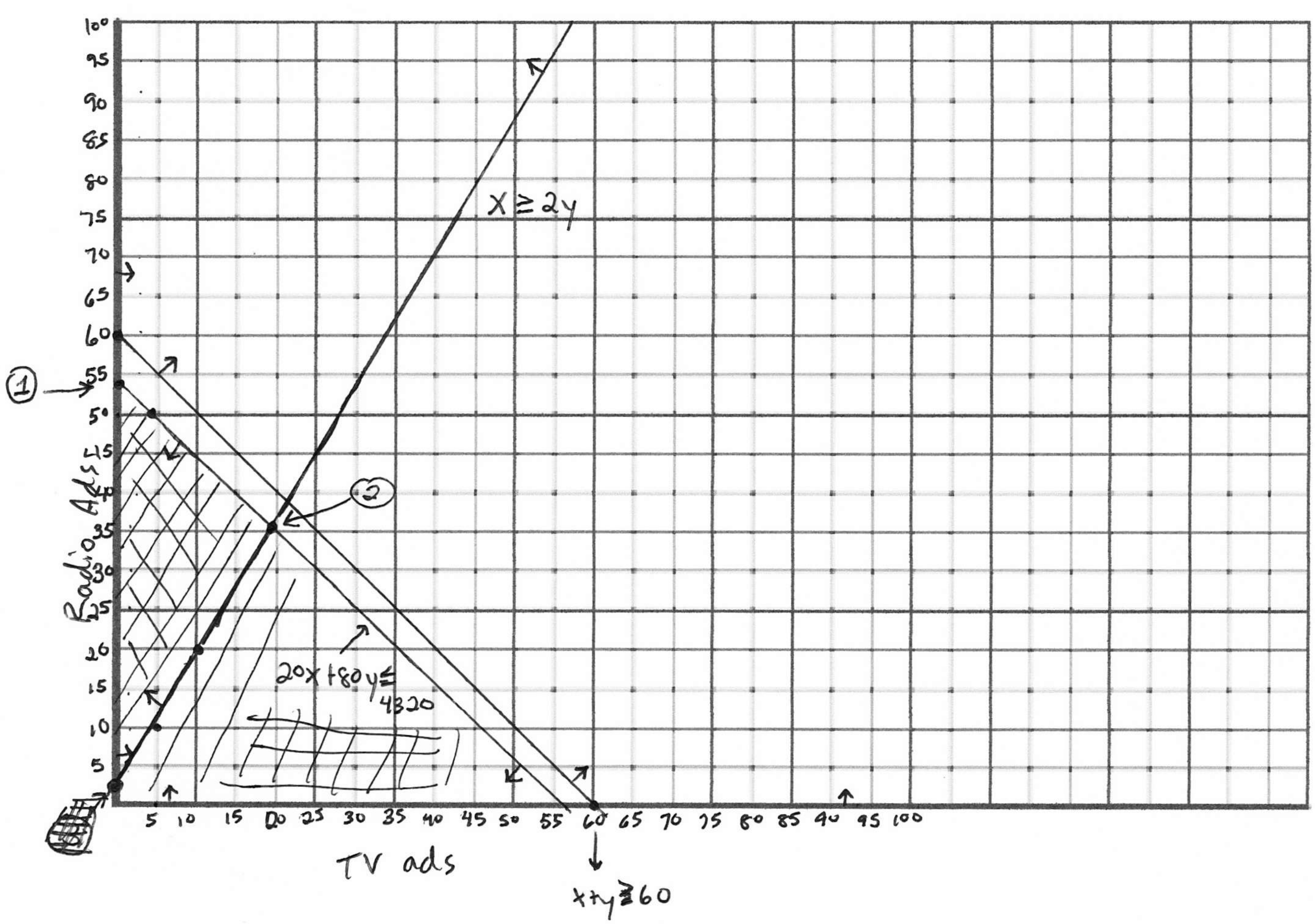
$X \geq \emptyset$
$Y \geq \emptyset$

5. Next, write down the function for the number of people that will be exposed to the advertisements. This is the Objective Function for the problem.

$$P = 2000X + 1500Y$$

You now have four linear inequalities and an objective function. These together describe the situation. This combined set of inequalities and objective function make up what is known mathematically as a **linear programming** problem. Write all of the inequalities and the objective function together below. This is typically written as a list of constraints, with the objective function last.

6. To solve this problem, you will need to graph the **intersection** of all five inequalities on one common XY plane. Do this on the grid below. Have the bottom left be the origin, with the horizontal axis representing X and the vertical axis representing Y. Label the axes with what they represent and label your lines as you graph them.



7. The shaded region in the above graph is called the feasible region. Any (x, y) point in the region corresponds to a possible number of radio and TV ads that will meet all the requirements of the problem. However, the values that will maximize the number of people exposed to the ads will occur at one of the vertices or corners of the region. Your region should have three corners. Find the coordinates of these corners by solving the appropriate system of linear equations. Be sure to *show your work* and *label* the (x, y) coordinates of the corners in your graph.

1 (0, 54) is the mavimum number of Radio ads that could be purchased

2 $\boxed{(20, 35)}$ 55 ads woud be the highest mixture of both ads that could be purchased.

3 (55, 0) is the maximum number of TV ads that could be purchased.

8. To find which number of radio and TV ads will maximize the number of people who are exposed to the business advertisements, evaluate the objective function P for each of the vertices you found. Show your work.

$x = 20$ $y = 35$ is the optimal number of ads for each area.

$2000(20) + 1500(35) = P$

$40{,}000 + 52{,}500 = \boxed{92{,}500}$

92,500 would be the amount of people exposed to all of the advertising.

9. Write a sentence describing how many of each type of advertisement should be purchased and what is the maximum number of people who will be exposed to the ad.

92,500 people would be exposed to the total advertisements if 20 TV ads and 35 radio ads were purchased.

10. Reflective Writing.

Did this project change the way you think about how math can be applied to the real world? Write one paragraph stating what ideas changed and why. If this project did not change the way you think, write how this project gave further evidence to support your existing opinion about applying math. Be specific.

I can see how this model would be extremely useful for a business to quickly and effectively evaluate a budget. It is fascinating to see how a few equations can tell such a detailed story and give such a broad picture for financial decisions.